Kola u ustaljenom prostoperiodičnom režimu

- svi naponi i sve struje u kolu su prostoperiodične (sinusoidalne ili kosinusoidalne) funkcije vremena sa istom kružnom učestanošću i u opštem slučaju različitim fazama
- $x(t) = X_m \cos(\omega t + \theta)$, gde su:
  - $x(t)$ – struja ili napon u vremenskom domenu
  - X_m – amplituda
  - ω – kružna učestanost
  - θ – faza
  - t – vremenska promenljiva
- važi jednakost: $\omega = 2\pi f = \frac{2\pi}{T}$, gde je:
  - f – frekvencija
  - T – perioda ($T = \frac{1}{f}$)

Fazori

34. Dati su vremenski oblici napona i struja. Odrediti odgovarajuće fazore.

a) $u_1(t) = 24\sqrt{2}\text{V}\cos(377t - 45°)$.

b) $i_2(t) = 12\text{A}\cos(377t + 120°)$.

c) $u_3(t) = 12\sqrt{2}\text{V}\cos(377t - 425°)$.

d) $i_4(t) = 18\sqrt{2}\text{A}\sin(2513t + 4{,}2°)$.

e) $u_5(t) = 2\text{V}\sin(2\pi ft), \quad f = 50\text{kHz}$.

f) $i_6(t) = -8\text{A}\sin(2\pi ft - 135°), \quad f = 20\text{kHz}$.

Rešenje:

a) $u_1(t) = 24\sqrt{2}\text{V}\cos(377t - 45°) \quad \Rightarrow \quad \boxed{\underline{U}_1 = 24\text{V}\angle -45°}$

b) $i_2(t) = 12\text{A}\cos(377t + 120°) \quad \Rightarrow \quad \boxed{\underline{I}_2 = 6\sqrt{2}\text{A}\angle 120°}$

c) $u_3(t) = 12\sqrt{2}\text{V}\cos(377t - 425°) \quad \Rightarrow \quad \boxed{\underline{U}_3 = 12\text{V}\angle -425° = 12\text{V}\angle -65°}$

d) $i_4(t) = 18\sqrt{2}\text{A}\sin(2513t + 4{,}2°) = 18\sqrt{2}\text{A}\cos(90° - (2513t + 4{,}2°))$

$i_4(t) = 18\sqrt{2}\text{A}\cos(-2513t + 85{,}8°) = 18\sqrt{2}\text{A}\cos(2513t - 85{,}8°)$

$\boxed{\underline{I}_4 = 18\text{A}\angle -85{,}8°}$

e) $u_5(t) = 2\text{V}\sin(2\pi ft) = 2\text{V}\cos(90° - 2\pi ft) = 2\text{V}\cos(2\pi ft - 90°)$

$\boxed{\underline{U}_5 = \sqrt{2}\text{V}\angle -90°}$

f) $i_6(t) = -8\text{A}\sin(2\pi ft - 135°) = 8\text{A}\sin(2\pi ft - 135° + 180°) = 8\text{A}\sin(2\pi ft + 45°)$

$i_6(t) = 8\text{A}\cos(90° - (2\pi ft + 45°)) = 8\text{A}\cos(-2\pi ft + 45°) = 8\text{A}\cos(2\pi ft - 45°)$

$\boxed{\underline{I}_6 = 4\sqrt{2}\text{A}\angle -45°}$

35. Konvertovati date fazore u vremenske oblike odgovarajućih napona i struja ako je poznato da je $f = 60\text{Hz}$.

a) $\underline{U}_1 = 16\text{V}\angle 20°$.

b) $\underline{I}_2 = 10\text{A}\angle -75°$.

Rešenje:

a) $\underline{U_1} = 16\text{V}\angle 20° \quad \Rightarrow \quad u_1(t) = 16\sqrt{2}\text{V}\cos(2\pi ft + 20°) \quad \Rightarrow \quad \boxed{u_1(t) = 16\sqrt{2}\text{V}\cos(120\pi t + 20°)}$

b) $\underline{I_2} = 10\text{A}\angle -75° \quad \Rightarrow \quad i_2(t) = 10\sqrt{2}\text{A}\cos(2\pi ft - 75°) \quad \Rightarrow \quad \boxed{i_2(t) = 10\sqrt{2}\text{A}\cos(120\pi t - 75°)}$

36. Konvertovati date fazore u vremenske oblike odgovarajućih napona i struja ako je poznato da je $f = 400\text{Hz}$.

a) $\underline{U_1} = 10\text{V}\angle 120°$.

b) $\underline{I_2} = 12\text{A}\angle -60°$.

Rešenje:

a) $\underline{U_1} = 10\text{V}\angle 120° \quad \Rightarrow \quad u_1(t) = 10\sqrt{2}\text{V}\cos(2\pi ft + 120°) \quad \Rightarrow \quad \boxed{u_1(t) = 10\sqrt{2}\text{V}\cos(800\pi t + 120°)}$

b) $\underline{I_2} = 12\text{A}\angle -60° \quad \Rightarrow \quad i_2(t) = 12\sqrt{2}\text{A}\cos(2\pi ft - 60°) \quad \Rightarrow \quad \boxed{i_2(t) = 12\sqrt{2}\text{A}\cos(800\pi t - 60°)}$

Konverzija napona i struja iz vremenskog domena u kompleksni (frekventni) domen i obratno

- konverzija napona i struja iz vremenskog domena ($x(t) = X_m\cos(\omega t + \theta)$) u kompleksni (frekventni) domen ($\underline{X} = a + jb$):

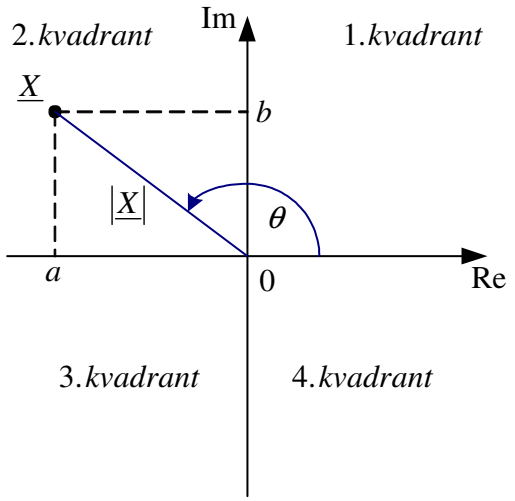
  $x(t) = X_m\cos(\omega t + \theta) \quad \rightarrow \quad fazor\ \frac{X_m}{\sqrt{2}}\angle\theta \quad \rightarrow \quad |\underline{X}|\angle\theta \quad \rightarrow \quad \underline{X} = |\underline{X}|\cdot e^{j\theta}$, gde je:

  $|\underline{X}| = \frac{X_m}{\sqrt{2}}$

  $\underline{X} = |\underline{X}|\cdot e^{j\theta} = |\underline{X}|\cdot(\cos\theta + j\sin\theta) = |\underline{X}|\cos\theta + j|\underline{X}|\sin\theta = a + jb$, gde su:

  $a = |\underline{X}|\cos\theta$, $b = |\underline{X}|\sin\theta$, $j -$ imaginarna jedinica ($j^2 = -1$).

- konverzija napona i struja iz kompleksnog (frekventnog) domena ($\underline{X} = a + jb$) u vremenski domen ($x(t) = X_m \cos(\omega t + \theta)$):



$\underline{X} = a + jb = |\underline{X}| \cdot e^{j\theta}$, pri čemu je:

$$|\underline{X}| = \sqrt{a^2 + b^2}$$

$$\theta = \begin{cases} arctg\dfrac{b}{a}; & za\ 1.\ i\ 4.\ kvadrant \\ 180° + arctg\dfrac{b}{a}; & za\ 2.\ i\ 3.\ kvadrant \end{cases}$$

Dalje se dobija:

$$|\underline{X}| \cdot e^{j\theta} \quad \rightarrow \quad fazor\ |\underline{X}| \angle \theta \quad \rightarrow \quad x(t) = X_m \cos(\omega t + \theta)$$

pri čemu je: $X_m = |\underline{X}|\sqrt{2}$.

37. Konvertovati date napone i struje iz vremenskog u kompleksni domen ako je poznato da je $f = 5\text{kHz}$.

a) $u_1(t) = 6\text{V}\cos(2\pi f t)$.

b) $u_2(t) = 2\text{V}\cos(2\pi f t + 45°)$.

c) $i_3(t) = \sqrt{2}\text{A}\sin(2\pi f t)$.

d) $i_4(t) = -3\sqrt{2}\text{A}\cos(2\pi f t)$.

<u>Rešenje:</u>

a) $u_1(t) = 6\text{V}\cos(2\pi f t) \quad \rightarrow \quad \underline{U_1} = 3\sqrt{2}\text{V} \angle 0° \quad \rightarrow \quad \underline{U_1} = 3\sqrt{2}\text{V} \cdot e^{j0°} = 3\sqrt{2}\text{V} \cdot (\cos 0° + j \sin 0°)$

$\underline{U_1} = 3\sqrt{2}\text{V} \cdot (1 + j0) \quad \Rightarrow \quad \boxed{\underline{U_1} = 3\sqrt{2}\text{V}}$

b)

$u_2(t) = 2\text{V}\cos(2\pi ft + 45°) \quad \rightarrow \quad \underline{U_2} = \sqrt{2}\text{V}\angle 45° \quad \rightarrow \quad \underline{U_2} = \sqrt{2}\text{V} \cdot e^{j45°} = \sqrt{2}\text{V} \cdot (\cos 45° + j \sin 45°)$

$\underline{U_2} = \sqrt{2}\text{V} \cdot \left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} \right) \quad \Rightarrow \quad \boxed{\underline{U_2} = (1 + j)\text{V}}$

c) $i_3(t) = \sqrt{2}\text{A}\sin(2\pi ft) = \sqrt{2}\text{A}\cos(90° - 2\pi ft) = \sqrt{2}\text{A}\cos(2\pi ft - 90°)$

$i_3(t) = \sqrt{2}\text{A}\cos(2\pi ft - 90°) \quad \rightarrow \quad \underline{I_3} = 1\text{A}\angle -90° \quad \rightarrow \quad \underline{I_3} = 1\text{A} \cdot e^{-j90°}$

$\underline{I_3} = 1\text{A} \cdot (\cos(-90°) + j \sin(-90°))$

$\underline{I_3} = 1\text{A} \cdot (0 - j1) \quad \Rightarrow \quad \boxed{\underline{I_3} = -j\text{A}}$

d) $i_4(t) = -3\sqrt{2}\text{A}\cos(2\pi ft) \quad \rightarrow \quad \underline{I_4} = -3\text{A}\angle 0° \quad \rightarrow \quad \underline{I_4} = -3\text{A} \cdot e^{j0°} = -3\text{A} \cdot (\cos 0° + j \sin 0°)$

$\underline{I_4} = -3\text{A} \cdot (1 + j0) \quad \Rightarrow \quad \boxed{\underline{I_4} = -3\text{A}}$

38. Konvertovati date napone i struje iz vremenskog u kompleksni domen ako je poznato da je $\omega = 6\text{krad/s}$.

a) $u_1(t) = -\sqrt{2}\text{V}\sin(\omega t - 150°)$.

b) $i_2(t) = 2\text{A}\sin(\omega t - 135°)$.

Rešenje:

a) $u_1(t) = -\sqrt{2}\text{V}\sin(\omega t - 150°) = -\sqrt{2}\text{V}\cos(90° - (\omega t - 150°)) = -\sqrt{2}\text{V}\cos(240° - \omega t)$

$u_1(t) = -\sqrt{2}\text{V}\cos(240° - \omega t) = -\sqrt{2}\text{V}\cos(\omega t - 240°) \quad \rightarrow \quad \underline{U_1} = -1\text{V}\angle -240° \quad \rightarrow \quad \underline{U_1} = -1\text{V} \cdot e^{-j240°}$

$\underline{U_1} = -1\text{V} \cdot (\cos(-240°) + j \sin(-240°))$

$\underline{U_1} = -1\text{V} \cdot \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \quad \Rightarrow \quad \boxed{\underline{U_1} = \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} \right)\text{V}}$

b) $i_2(t) = 2\text{A}\sin(\omega t - 135°) = 2\text{A}\cos(90° - (\omega t - 135°)) = 2\text{A}\cos(225° - \omega t)$

$i_2(t) = 2\text{A}\cos(225° - \omega t) = 2\text{A}\cos(\omega t - 225°) \quad \rightarrow \quad \underline{I_2} = \sqrt{2}\text{A}\angle -225° \quad \rightarrow \quad \underline{I_2} = \sqrt{2}\text{A}\cdot e^{-j225°}$

$\underline{I_2} = \sqrt{2}\text{A}\cdot\left(\cos(-225°) + j\sin(-225°)\right)$

$$\underline{I_2} = \sqrt{2}\text{A}\cdot\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) \quad \Rightarrow \quad \boxed{\underline{I_2} = (-1 + j)\text{A}}$$

39. Konvertovati date napone i struje iz kompleksnog u vremenski domen ako je poznato da je $f = 2\text{kHz}$.

a) $\underline{U_1} = (5 + j5)\text{V}$.

b) $\underline{I_2} = (-3 + j4)\text{A}$.

<u>Rešenje:</u>

a) $\underline{U_1} = (5 + j5)\text{V} \quad \rightarrow \quad \underline{U_1} = \sqrt{5^2 + 5^2}\,\text{V}\cdot e^{jarctg\frac{5}{5}} = 5\sqrt{2}\text{V}\cdot e^{j45°} \quad \rightarrow \quad \underline{U_1} = 5\sqrt{2}\text{V}\angle 45°$

$\underline{U_1} = 5\sqrt{2}\text{V}\angle 45° \quad \rightarrow \quad u_1(t) = 10\text{V}\cos(2\pi ft + 45°) \quad \Rightarrow \quad \boxed{u_1(t) = 10\text{V}\cos(4000\pi t + 45°)}$

b) $\underline{I_2} = (-3 + j4)\text{A} \quad \rightarrow \quad \underline{I_2} = \sqrt{(-3)^2 + 4^2}\,\text{A}\cdot e^{j\left(180° + arctg\frac{4}{(-3)}\right)} = 5\text{A}\cdot e^{j(180° - 53{,}13°)} = 5\text{A}\cdot e^{j126{,}87°}$

$\underline{I_2} = 5\text{A}\cdot e^{j126{,}87°} \quad \rightarrow \quad \underline{I_2} = 5\text{A}\angle 126{,}87°$

$\underline{I_2} = 5\text{A}\angle 126{,}87° \rightarrow i_2(t) = 5\sqrt{2}\text{A}\cos(2\pi ft + 126{,}87°) \quad \Rightarrow \quad \boxed{i_2(t) = 5\sqrt{2}\text{A}\cos(4000\pi t + 126{,}87°)}$

40. Konvertovati date napone i struje iz kompleksnog u vremenski domen ako je poznato da je $\omega = 10\text{krad/s}$.

a) $\underline{U_1} = (-7 - j2)\text{V}$.

b) $\underline{I_2} = (2 - j5)\text{A}$.

Rešenje:

a)

$$\underline{U}_1 = (-7 - j2)\text{V} \quad \rightarrow \quad \underline{U}_1 = \sqrt{(-7)^2 + (-2)^2}\,\text{V} \cdot e^{j\left(180° + arctg\frac{(-2)}{(-7)}\right)} = \sqrt{53}\text{V} \cdot e^{j(180° + 15{,}95°)} = \sqrt{53}\text{V} \cdot e^{j195{,}95°}$$

$$\underline{U}_1 = \sqrt{53}\text{V} \cdot e^{j195{,}95°} \quad \rightarrow \quad \underline{U}_1 = \sqrt{53}\text{V}\angle 195{,}95°$$

$$\underline{U}_1 = \sqrt{53}\text{V}\angle 195{,}95° \rightarrow u_1(t) = \sqrt{106}\text{V}\cos(\omega t + 195{,}95°) \Rightarrow \boxed{u_1(t) = \sqrt{106}\text{V}\cos(10000t + 195{,}95°)}$$

b) $\underline{I}_2 = (2 - j5)\text{A} \quad \rightarrow \quad \underline{I}_2 = \sqrt{2^2 + (-5)^2}\,\text{A} \cdot e^{jarctg\frac{(-5)}{2}} = \sqrt{29}\text{A} \cdot e^{-j68{,}2°} \quad \rightarrow \quad \underline{I}_2 = \sqrt{29}\text{A}\angle -68{,}2°$

$$\underline{I}_2 = \sqrt{29}\text{A}\angle -68{,}2° \rightarrow i_2(t) = \sqrt{58}\text{A}\cos(\omega t - 68{,}2°) \Rightarrow \boxed{i_2(t) = \sqrt{58}\text{A}\cos(10000t - 68{,}2°)}$$

Rešavanje kola u ustaljenom prostoperiodičnom režimu

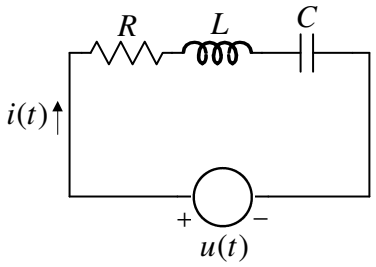
Rešavanje kola u ustaljenom prostoperiodičnom režimu se vrši u četiri koraka:

- konverzija svih napona i struja iz vremenskog u kompleksni domen
- konverzija otpornosti otpornika, induktivnosti kalemova i kapacitivnosti kondenzatora u odgovarajuće impedanse ($\underline{Z_R} = R$, $\underline{Z_L} = j\omega L$, $\underline{Z_C} = \frac{1}{j\omega C}$)
- rešavanje kola u kompleksnom domenu primenom zakona i metoda koje su rađene kod kola sa jednosmernim strujama
- konverzija dobijenog rezultata (struje i/ili napona) iz kompleksnog u vremenski domen

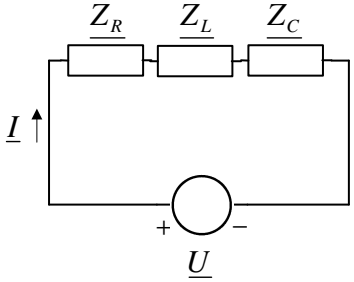
41. Za kolo sa slike je poznato $u(t) = 50\sqrt{2}\text{V}\cos(\omega t + 30°)$, $R = 25\Omega$, $L = 20\text{mH}$ i $C = 50\mu\text{F}$.

a) Izračunati ekvivalentnu impedansu tri redno vezana elementa (otpornika, kalema i kondenzatora), kao i struju $i(t)$ ako kolo radi na frekvenciji $f = 60\text{Hz}$.

b) Izračunati ekvivalentnu impedansu tri redno vezana elementa (otpornika, kalema i kondenzatora) ako kolo radi na frekvenciji $f = 400\text{Hz}$.



Rešenje:



a) $\underline{Z_R} = R = 25\Omega$

$\underline{Z_L} = j\omega L = j \cdot 2\pi \cdot 60\text{Hz} \cdot 20\text{mH} = j7{,}54\Omega$

$\underline{Z_C} = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C} = -\dfrac{j}{2\pi \cdot 60\text{Hz} \cdot 50\mu\text{F}} = -j53{,}05\Omega$

$u(t) = 50\sqrt{2}\text{V}\cos(\omega t + 30°) \quad \rightarrow \quad \underline{U} = 50\text{V}\angle 30° \quad \rightarrow \quad \underline{U} = 50\text{V} \cdot e^{j30°} = 50\text{V} \cdot (\cos 30° + j \sin 30°)$

$\underline{U} = 50\text{V} \cdot \left(\dfrac{\sqrt{3}}{2} + j\dfrac{1}{2} \right) \quad \Rightarrow \quad \underline{U} = 25(\sqrt{3} + j)\text{V}$

$\underline{Z_{EKV}} = \underline{Z_R} + \underline{Z_C} + \underline{Z_L} \quad \Rightarrow \quad \boxed{\underline{Z_{EKV}} = (25 - j45{,}51)\Omega}$

$\underline{I} = \dfrac{\underline{U}}{\underline{Z_{EKV}}} = \dfrac{25(\sqrt{3} + j)\text{V}}{(25 - j45{,}51)\Omega} = \dfrac{25(\sqrt{3} + j)\text{V}}{(25 - j45{,}51)\Omega} \cdot \dfrac{(25 + j45{,}51)}{(25 + j45{,}51)} = \left(\dfrac{-55{,}22 + j2595{,}64}{2696{,}16} \right)\text{A}$

$\underline{I} = (-0{,}0205 + j0{,}9627)\text{A} \quad \rightarrow \quad \underline{I} = \sqrt{(-0{,}0205)^2 + 0{,}9627^2}\,\text{A} \cdot e^{j\left(180° + arctg\frac{0{,}9627}{(-0{,}0205)}\right)}$

$\underline{I} = 0{,}96\text{A} \cdot e^{j(180° - 88{,}78°)} = 0{,}96\text{A} \cdot e^{j91{,}22°} \quad \rightarrow \quad \underline{I} = 0{,}96\text{A}\angle 91{,}22°$

$\underline{I} = 0{,}96\text{A}\angle 91{,}22° \rightarrow i(t) = 0{,}96\sqrt{2}\text{A}\cos(2\pi f t + 91{,}22°) \Rightarrow \boxed{i(t) = 1{,}36\text{A}\cos(120\pi t + 91{,}22°)}$

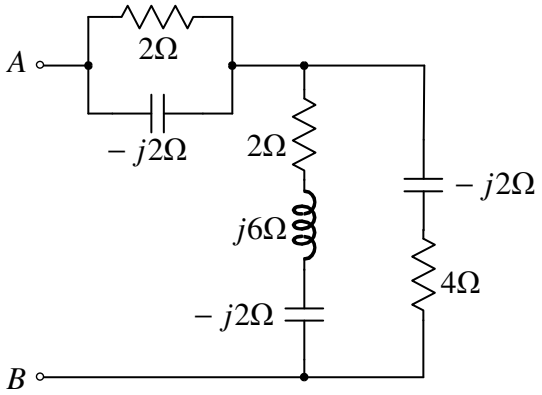
b) $\underline{Z_R} = R = 25\Omega$

$\underline{Z_L} = j\omega L = j \cdot 2\pi \cdot 400\text{Hz} \cdot 20\text{mH} = j50{,}27\Omega$

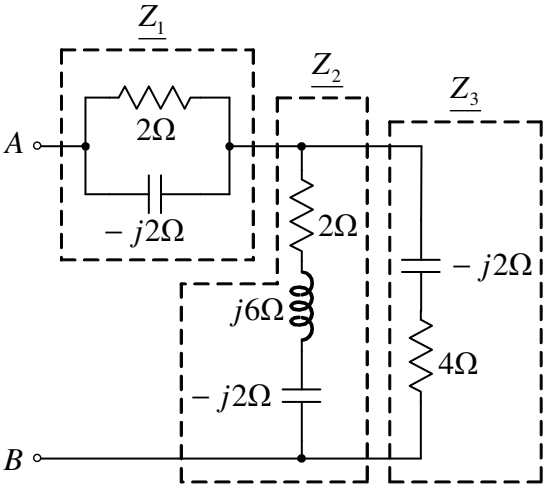
$\underline{Z_C} = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C} = -\dfrac{j}{2\pi \cdot 400\text{Hz} \cdot 50\mu\text{F}} = -j7{,}96\Omega$

$$\underline{Z_{EKV}} = \underline{Z_R} + \underline{Z_C} + \underline{Z_L} \quad \Rightarrow \quad \boxed{\underline{Z_{EKV}} = (25 + j42{,}31)\Omega}$$

42. Za kolo sa slike odrediti ekvivalentnu impedansu između tačaka A i B.



Rešenje:

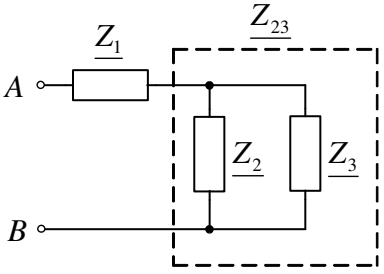


$$\underline{Z_1} = 2\Omega \parallel (-j2\Omega) = \frac{2\cdot(-j2)}{2-j2}\Omega = \frac{-j4}{2-j2}\Omega \cdot \frac{2+j2}{2+j2}$$

$$\underline{Z_1} = \frac{8-j8}{8}\Omega = (1-j)\Omega$$

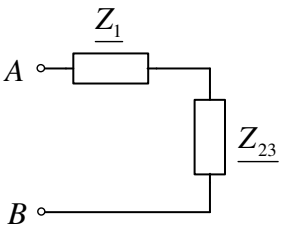
$$\underline{Z_2} = 2 + j6 - j2 = (2+j4)\Omega$$

$$\underline{Z_3} = (4-j2)\Omega$$



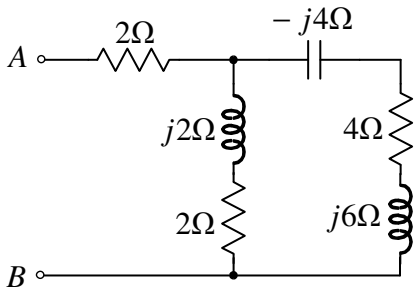
$$\underline{Z_{23}} = \underline{Z_2} \parallel \underline{Z_3} = \frac{\underline{Z_2}\cdot\underline{Z_3}}{\underline{Z_2}+\underline{Z_3}} = \frac{(2+j4)\cdot(4-j2)}{(2+j4)+(4-j2)}\Omega = \frac{16+j12}{6+j2}\Omega = \frac{8+j6}{3+j}\Omega$$

$$\underline{Z_{23}} = \frac{8+j6}{3+j}\Omega \cdot \frac{3-j}{3-j} = (3+j)\Omega$$



$$\underline{Z_{EKV}} = \underline{Z_1} + \underline{Z_{23}} = (1 - j + 3 + j)\Omega \quad \Rightarrow \quad \boxed{\underline{Z_{EKV}} = 4\Omega}$$

43. (Zadatak za vežbu) Za kolo sa slike odrediti ekvivalentnu impedansu između tačaka A i B.

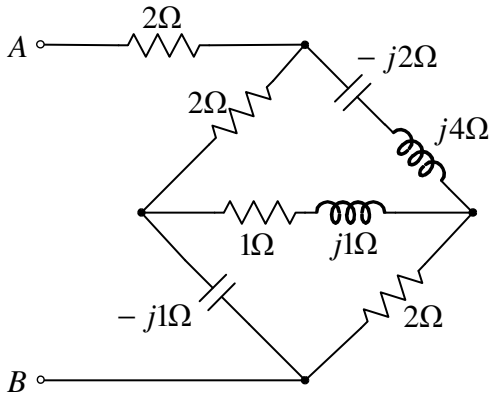


Rešenje:

$$\boxed{\underline{Z_{EKV}} = (3{,}38 + j1{,}08)\Omega}$$

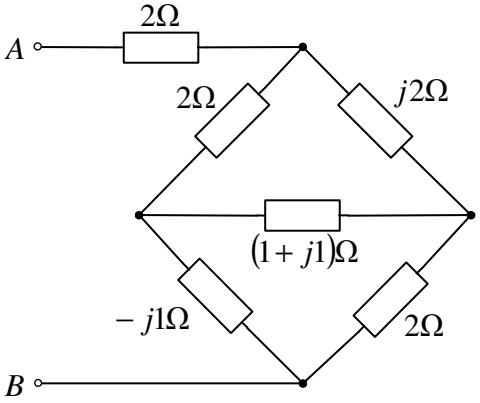
44. a) Za kolo sa slike odrediti ekvivalentnu impedansu između tačaka A i B.

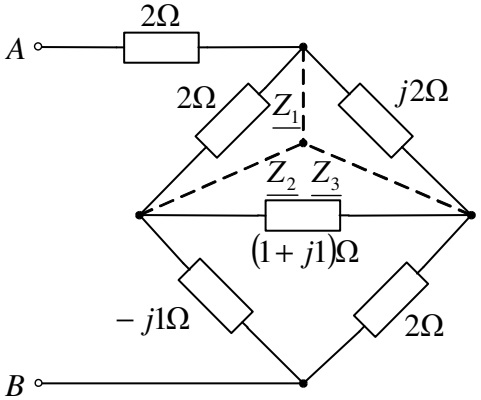
b) Ako je $f = 50\text{Hz}$, realizovati dobijenu ekvivalentnu impedansu korišćenjem minimalnog broja pasivnih komponenata (otpornika i/ili kalemova i/ili kondenzatora) i odrediti njihove vrednosti.



Rešenje:

a)

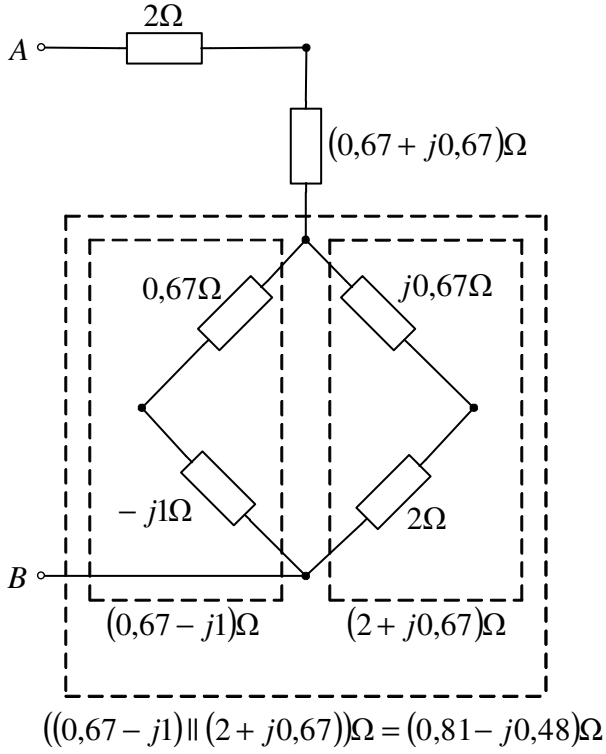




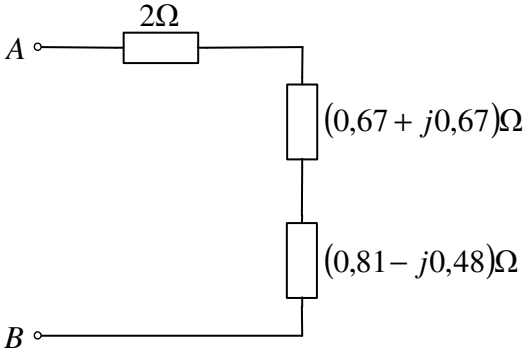
$$\underline{Z_1} = \frac{2\Omega \cdot j2\Omega}{2\Omega + j2\Omega + (1+j)\Omega} = \frac{j4}{3+j3}\Omega = \frac{j4}{3+j3}\cdot\frac{3-j3}{3-j3}\Omega = \frac{12+j12}{18}\Omega = (0{,}67 + j0{,}67)\Omega$$

$$\underline{Z_2} = \frac{2\Omega \cdot (1+j)\Omega}{2\Omega + j2\Omega + (1+j)\Omega} = \frac{2+j2}{3+j3}\Omega = 0{,}67\Omega$$

$$\underline{Z_3} = \frac{j2\Omega \cdot (1+j)\Omega}{2\Omega + j2\Omega + (1+j)\Omega} = \frac{-2+j2}{3+j3}\Omega = \frac{2}{3}\cdot\frac{-1+j}{1+j}\cdot\frac{1-j}{1-j}\Omega = -\frac{2}{3}\cdot\frac{-2j}{2}\Omega = j0{,}67\Omega$$



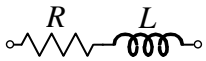
$$((0{,}67 - j1) \parallel (2 + j0{,}67))\Omega = (0{,}81 - j0{,}48)\Omega$$



$$\underline{Z_{EKV}} = (2 + 0{,}67 + j0{,}67 + 0{,}81 - j0{,}48)\Omega$$

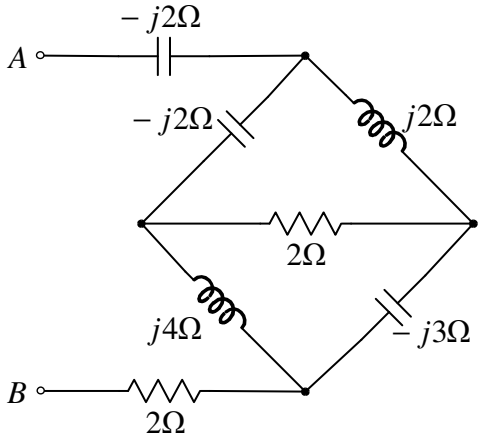
$$\boxed{\underline{Z_{EKV}} = (3{,}48 + j0{,}19)\Omega}$$

b) $\underline{Z_{EKV}} = (3{,}48 + j0{,}19)\Omega = R + j\omega L = R + j2\pi fL \quad \Rightarrow \quad \boxed{R = 3{,}48\Omega} \qquad \boxed{L = 604{,}8\mu\text{H}}$



45. (Zadatak za vežbu) a) Za kolo sa slike odrediti ekvivalentnu impedansu između tačaka A i B.

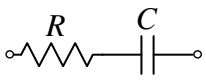
b) Ako je $f = 100\text{Hz}$, realizovati dobijenu ekvivalentnu impedansu korišćenjem minimalnog broja pasivnih komponenata (otpornika i/ili kalemova i/ili kondenzatora) i odrediti njihove vrednosti.



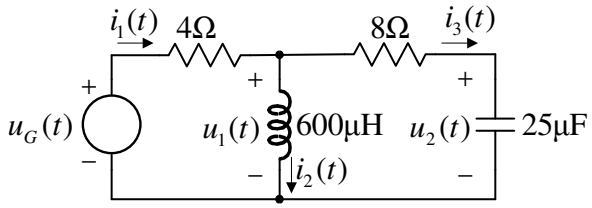
Rešenje:

a) $\boxed{\underline{Z_{EKV}} = (4 - j4)\Omega}$

b) $\underline{Z_{EKV}} = (4 - j4)\Omega = R - \dfrac{j}{\omega C} = R - \dfrac{j}{2\pi fC} \quad \Rightarrow \quad \boxed{R = 4\Omega} \qquad \boxed{C = 398\mu\text{F}}$



46. Direktnom primenom Kirhofovih zakona i Omovog zakona odrediti napone $u_1(t)$, $u_2(t)$ i struje $i_1(t)$, $i_2(t)$ i $i_3(t)$ u kolu sa slike. Poznato je $u_G(t) = 24\sqrt{2}\text{V}\sin(\omega t + 150°)$ i $\omega = 10\text{krad/s}$.



Rešenje:

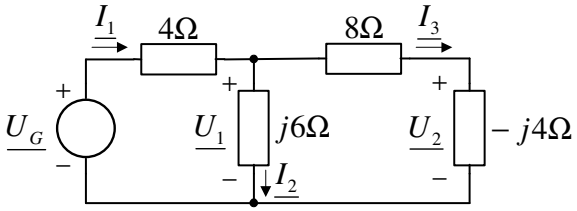
$$L = 600\mu\text{H} \quad \rightarrow \quad \underline{Z_L} = j\omega L = j \cdot 10\text{krad/s} \cdot 600\mu\text{H} = j6\Omega$$

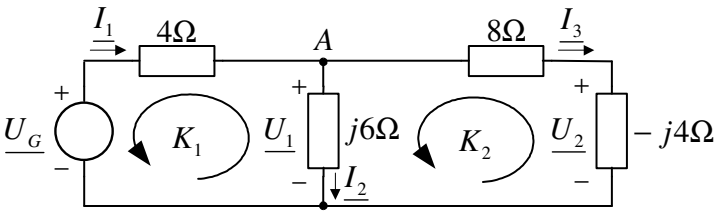
$$C = 25\mu\text{F} \quad \rightarrow \quad \underline{Z_C} = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -\frac{j}{10\text{krad/s} \cdot 25\mu\text{F}} = -j4\Omega$$

$$u_G(t) = 24\sqrt{2}\text{V}\sin(\omega t + 150°) = 24\sqrt{2}\text{V}\cos(90° - (\omega t + 150°)) = 24\sqrt{2}\text{V}\cos(-60° - \omega t)$$

$$u_G(t) = 24\sqrt{2}\text{V}\cos(\omega t + 60°) \quad \rightarrow \quad \underline{U_G} = 24\text{V}\angle 60° \quad \rightarrow \quad \underline{U_G} = 24\text{V} \cdot e^{j60°} = 24\text{V} \cdot (\cos 60° + j\sin 60°)$$

$$\underline{U_G} = 24\text{V} \cdot \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) \quad \Rightarrow \quad \underline{U_G} = (12 + j12\sqrt{3})\text{V}$$





Kontura K_1: $\underline{I_2} \cdot j6 + \underline{I_1} \cdot 4 - (12 + j12\sqrt{3}) = 0$

Kontura K_2: $\underline{I_3} \cdot (-j4) + \underline{I_3} \cdot 8 - \underline{I_2} \cdot j6 = 0$

Čvor A: $\underline{I_1} = \underline{I_2} + \underline{I_3}$

$$\left.\begin{aligned} \underline{I_1} &= \frac{1}{4} \cdot (12 + j12\sqrt{3} - \underline{I_2} \cdot j6) = 3 + j3\sqrt{3} - \underline{I_2} \cdot j\frac{3}{2} \\ \underline{I_3} &= \underline{I_2} \cdot \frac{j6}{8 - j4} = \underline{I_2} \cdot \frac{j6}{8 - j4} \cdot \frac{8 + j4}{8 + j4} = \underline{I_2} \cdot \frac{-3 + j6}{10} \\ \underline{I_1} &= \underline{I_2} + \underline{I_3} \end{aligned}\right\}$$

$$3 + j3\sqrt{3} - \underline{I}_2 \cdot j\frac{3}{2} = \underline{I}_2 - \frac{3}{10}\underline{I}_2 + j\frac{3}{5}\underline{I}_2 \quad \Rightarrow \quad \underline{I}_2 = \frac{30 \cdot (1 + j\sqrt{3})}{7 \cdot (1 + j3)} = \frac{30 \cdot (1 + j\sqrt{3})}{7 \cdot (1 + j3)} \cdot \frac{(1 - j3)}{(1 - j3)}$$

$$\underline{I}_2 = \frac{3}{7} \cdot (1 + 3\sqrt{3} + j(\sqrt{3} - 3)) = (2{,}655 - j0{,}543)\text{A} = 2{,}71\text{A}\angle -11{,}56°$$

$$\underline{I}_2 = 2{,}71\text{A}\angle -11{,}56° \quad \Rightarrow \quad \boxed{i_2(t) = 3{,}83\text{A}\cos(10000t - 11{,}56°)}$$

$$\underline{I}_1 = 3 + j3\sqrt{3} - j\frac{3}{2} \cdot (2{,}655 - j0{,}543) = (2{,}186 + j1{,}214)\text{A} = 2{,}5\text{A}\angle 29{,}05°$$

$$\underline{I}_1 = 2{,}5\text{A}\angle 29{,}05° \quad \Rightarrow \quad \boxed{i_1(t) = 3{,}54\text{A}\cos(10000t + 29{,}05°)}$$

$$\underline{I}_3 = (2{,}655 - j0{,}543) \cdot \frac{-3 + j6}{10} = (-0{,}471 + j1{,}756)\text{A} = 1{,}82\text{A}\angle 105{,}01°$$

$$\underline{I}_3 = 1{,}82\text{A}\angle 105{,}01° \quad \Rightarrow \quad \boxed{i_3(t) = 2{,}57\text{A}\cos(10000t + 105{,}01°)}$$

$$\underline{U}_1 = \underline{I}_2 \cdot j6 = (2{,}655 - j0{,}543) \cdot j6 = (3{,}258 + j15{,}93)\text{V} = 16{,}26\text{V}\angle 78{,}44°$$

$$\underline{U}_1 = 16{,}26\text{V}\angle 78{,}44° \quad \Rightarrow \quad \boxed{u_1(t) = 23\text{V}\cos(10000t + 78{,}44°)}$$

$$\underline{U}_2 = \underline{I}_3 \cdot (-j4) = (-0{,}471 + j1{,}756) \cdot (-j4) = (7{,}024 + j1{,}884)\text{V} = 7{,}27\text{V}\angle 15{,}01°$$

$$\underline{U}_2 = 7{,}27\text{V}\angle 15{,}01° \quad \Rightarrow \quad \boxed{u_2(t) = 10{,}28\text{V}\cos(10000t + 15{,}01°)}$$

47. Za kolo sa slike je poznato $u_G(t) = 12\sqrt{2}\text{V}\cos(2\pi ft)$, $i_G(t) = 2\sqrt{2}\text{A}\cos(2\pi ft)$ i $f = 5\text{kHz}$. Odrediti struju $i_0(t)$:

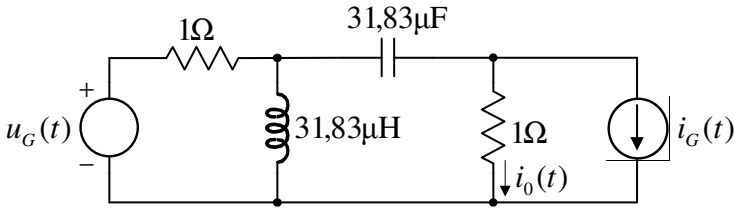
a) primenom metode potencijala čvorova;

b) primenom metode superpozicije;

c) primenom metode transformacije izvora;

d) primenom Tevenenove teoreme;

e) primenom Nortonove teoreme.



Rešenje:

$L = 31{,}83\mu\text{H} \quad \rightarrow \quad \underline{Z_L} = j\omega L = j \cdot 2\pi \cdot 5000\text{Hz} \cdot 31{,}83\mu\text{H} = j1\Omega$

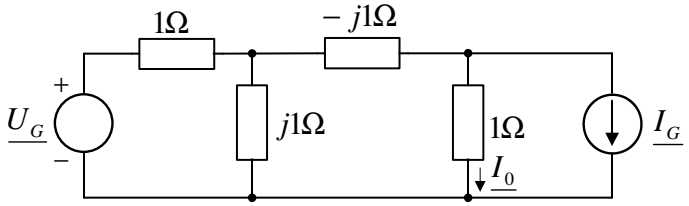
$C = 31{,}83\mu\text{F} \quad \rightarrow \quad \underline{Z_C} = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C} = -\dfrac{j}{2\pi \cdot 5000\text{Hz} \cdot 31{,}83\mu\text{F}} = -j1\Omega$

$u_G(t) = 12\sqrt{2}\text{V}\cos(2\pi ft) \quad \rightarrow \quad \underline{U_G} = 12\text{V}\angle 0° \quad \rightarrow \quad \underline{U_G} = 12\text{V} \cdot e^{j0°} = 12\text{V} \cdot (\cos 0° + j\sin 0°)$

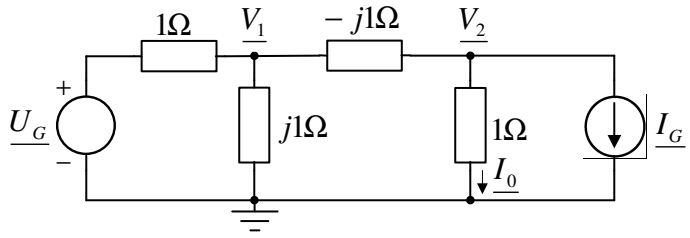
$\underline{U_G} = 12\text{V} \cdot (1 + j0) \quad \Rightarrow \quad \underline{U_G} = 12\text{V}$

$i_G(t) = 2\sqrt{2}\text{A}\cos(2\pi ft) \quad \rightarrow \quad \underline{I_G} = 2\text{A}\angle 0° \quad \rightarrow \quad \underline{I_G} = 2\text{A} \cdot e^{j0°} = 2\text{A} \cdot (\cos 0° + j\sin 0°)$

$\underline{I_G} = 2\text{A} \cdot (1 + j0) \quad \Rightarrow \quad \underline{I_G} = 2\text{A}$



a)



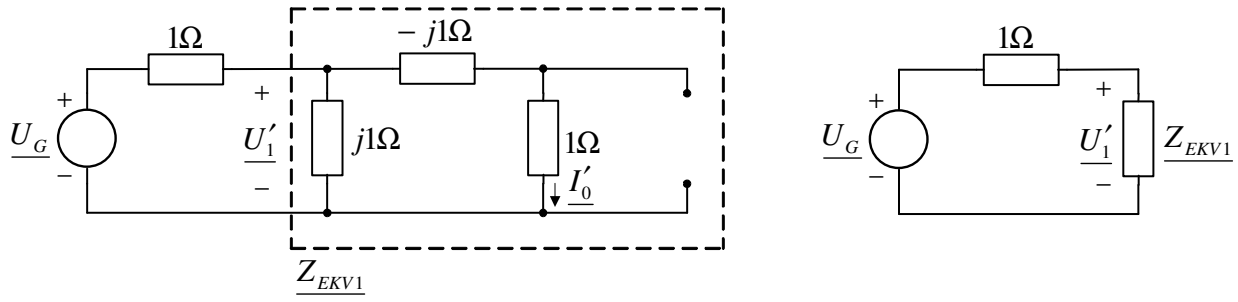
$$\left.\begin{aligned}\left(\frac{1}{1\Omega}+\frac{1}{j1\Omega}+\frac{1}{-j1\Omega}\right)\underline{V}_1-\left(\frac{1}{-j1\Omega}\right)\underline{V}_2=\frac{12\text{V}}{1\Omega}\\ -\left(\frac{1}{-j1\Omega}\right)\underline{V}_1+\left(\frac{1}{1\Omega}+\frac{1}{-j1\Omega}\right)\underline{V}_2=-2\text{A}\end{aligned}\right\}\quad\left.\begin{aligned}\underline{V}_1-j\underline{V}_2=12\\ -j\underline{V}_1+(1+j)\underline{V}_2=-2\end{aligned}\right\}\quad\left.\begin{aligned}\underline{V}_1=12+j\underline{V}_2\\ -j(12+j\underline{V}_2)+(1+j)\underline{V}_2=-2\end{aligned}\right\}$$

$$\underline{V}_2=\frac{-2+\text{j}12}{2+j}=\frac{-2+\text{j}12}{2+j}\cdot\frac{2-j}{2-j}=\left(\frac{8}{5}+j\frac{26}{5}\right)\text{V}=(1{,}6+j5{,}2)\text{V}$$

$$\underline{V}_1=12+j\underline{V}_2=\left(\frac{34}{5}+j\frac{8}{5}\right)\text{V}=(6{,}8+j1{,}6)\text{V}$$

$$\underline{I}_0=\frac{\underline{V}_2}{1\Omega}=(1{,}6+j5{,}2)\text{A}=5{,}44\text{A}\angle 72{,}9^\circ\quad\Rightarrow\quad\boxed{i_0(t)=7{,}69\text{A}\cos(10000\pi t+72{,}9^\circ)}$$

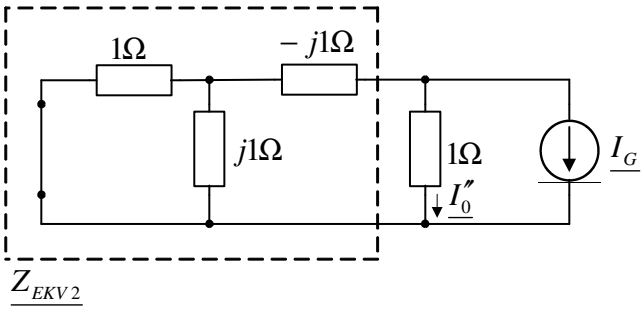
b)

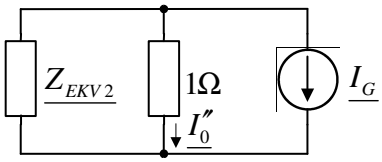


$$\underline{Z}_{EKV1}=j\parallel(1-j)=\frac{j(1-j)}{j+1-j}=(1+j)\Omega$$

$$\underline{U}_1'=\frac{\underline{Z}_{EKV1}}{\underline{Z}_{EKV1}+1\Omega}\cdot\underline{U}_G=\frac{1+j}{1+j+1}\cdot 12=\frac{1+j}{2+j}\cdot\frac{2-j}{2-j}\cdot 12=\frac{36+j12}{5}$$

$$\underline{I}_0'=\frac{\underline{U}_1'}{1-j}=\frac{36+j12}{5(1-j)}=\frac{36+j12}{5(1-j)}\cdot\frac{1+j}{1+j}=\left(\frac{12}{5}+j\frac{24}{5}\right)\text{A}$$





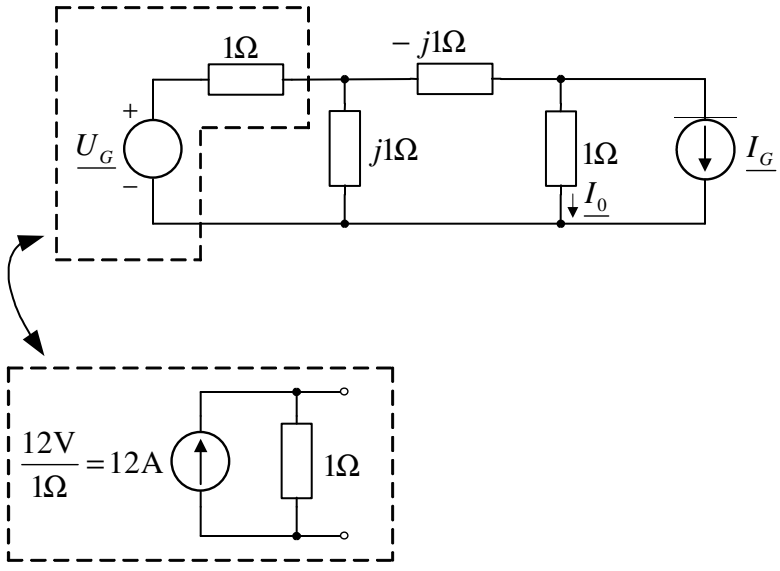
$$\underline{Z_{EKV2}} = -j + (1 \parallel j) = -j + \frac{j \cdot 1}{j+1} = \frac{1-j}{2}\,\Omega$$

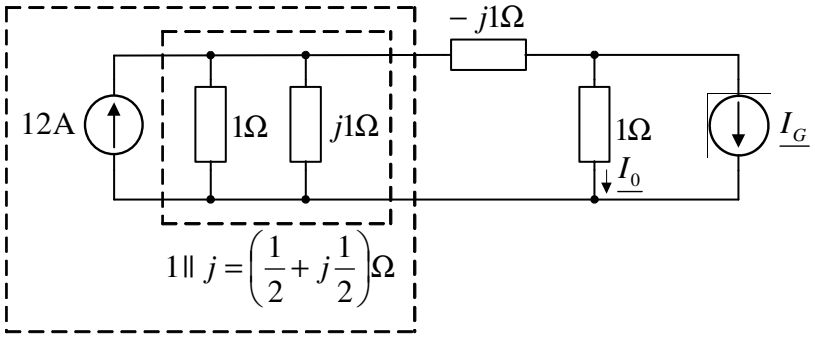
$$\underline{I_0''} = \frac{\underline{Z_{EKV2}}}{\underline{Z_{EKV2}} + 1\Omega} \cdot (-\underline{I_G}) = \frac{\frac{1-j}{2}}{\frac{1-j}{2}+1} \cdot (-2) = \frac{1-j}{3-j} \cdot (-2) = \frac{1-j}{3-j} \cdot (-2) \cdot \frac{3+j}{3+j} = \left(-\frac{4}{5} + j\frac{2}{5}\right)\mathrm{A}$$

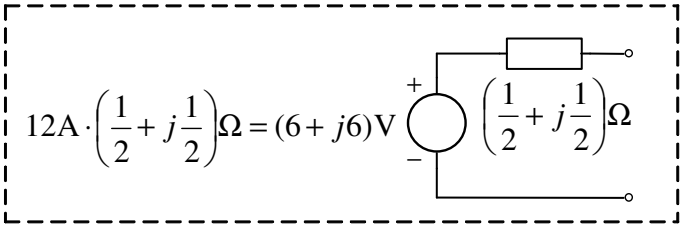
$$\underline{I_0} = \underline{I_0'} + \underline{I_0''} = \left(\frac{12}{5} + j\frac{24}{5}\right)\mathrm{A} + \left(-\frac{4}{5} + j\frac{2}{5}\right)\mathrm{A} = \left(\frac{8}{5} + j\frac{26}{5}\right)\mathrm{A}$$

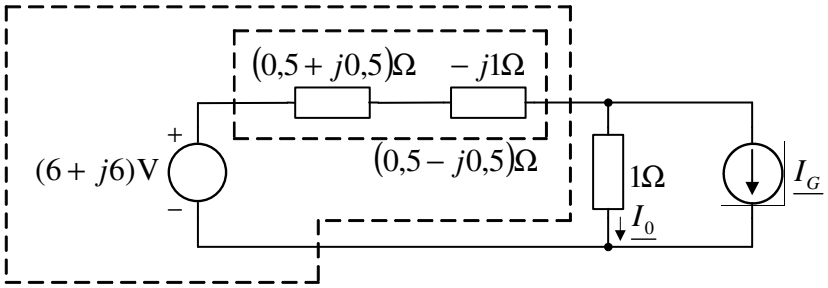
$$\underline{I_0} = (1{,}6 + j5{,}2)\mathrm{A} = 5{,}44\mathrm{A}\angle 72{,}9° \quad \Rightarrow \quad \boxed{i_0(t) = 7{,}69\mathrm{A}\cos(10000\pi t + 72{,}9°)}$$

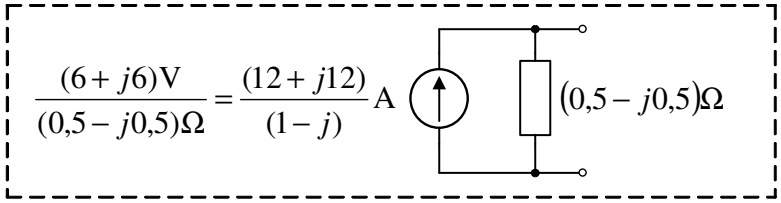
c)

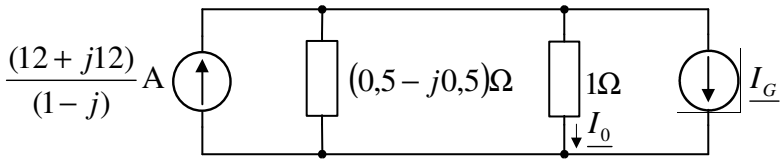


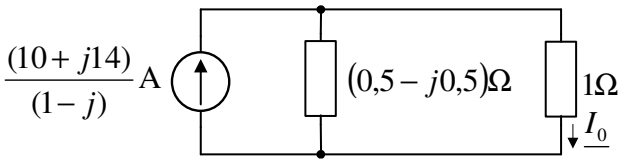

$-j1\Omega$
12A
1Ω
$j1\Omega$
1Ω
$\underline{I}_0$
$\underline{I}_G$
$1 \| j = \left(\frac{1}{2} + j\frac{1}{2}\right)\Omega$


$12\text{A} \cdot \left(\frac{1}{2} + j\frac{1}{2}\right)\Omega = (6 + j6)\text{V}$
+
−
$\left(\frac{1}{2} + j\frac{1}{2}\right)\Omega$


$(0{,}5 + j0{,}5)\Omega$
$-j1\Omega$
+
$(6 + j6)\text{V}$
−
$(0{,}5 - j0{,}5)\Omega$
1Ω
$\underline{I}_0$
$\underline{I}_G$


$\frac{(6 + j6)\text{V}}{(0{,}5 - j0{,}5)\Omega} = \frac{(12 + j12)}{(1 - j)}\text{A}$
$(0{,}5 - j0{,}5)\Omega$

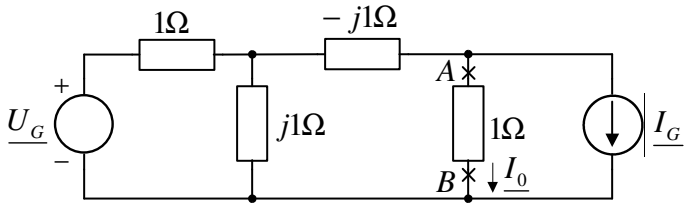

$\frac{(12 + j12)}{(1 - j)}\text{A}$
$(0{,}5 - j0{,}5)\Omega$
1Ω
$\underline{I}_0$
$\underline{I}_G$

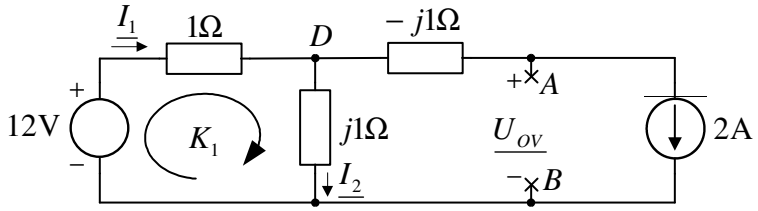


$$\underline{I}_0 = \frac{0{,}5-j0{,}5}{1+(0{,}5-j0{,}5)} \cdot \frac{10+j14}{1-j} = \left(\frac{8}{5}+j\frac{26}{5}\right)\text{A}$$

$$\underline{I}_0 = (1{,}6+j5{,}2)\text{A} = 5{,}44\text{A}\angle 72{,}9° \quad \Rightarrow \quad \boxed{i_0(t) = 7{,}69\text{A}\cos(10000\pi t + 72{,}9°)}$$

d)



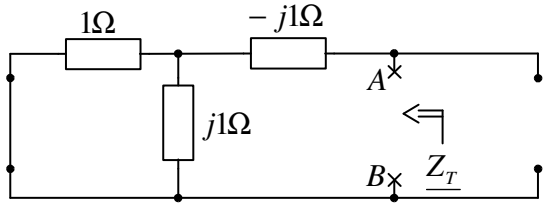


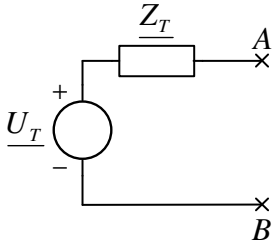
Kontura K_1: $\quad 12\text{V} - \underline{I}_1 \cdot 1\Omega - \underline{I}_2 \cdot j1\Omega = 0$

Čvor D: $\qquad \underline{I}_1 = \underline{I}_2 + 2\text{A}$

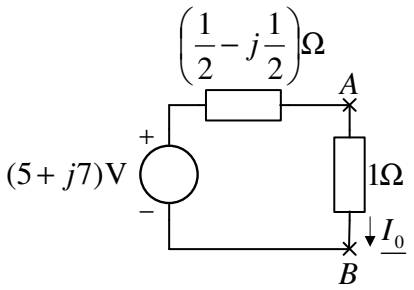
$$12 - \underline{I}_2 - 2 - \underline{I}_2 \cdot j = 0 \quad \Rightarrow \quad \underline{I}_2 = \frac{10}{1+j} = 5(1-j)\text{A}$$

$$\underline{U}_T = \underline{U}_{OV} = j \cdot \underline{I}_2 - (-j) \cdot 2 = j \cdot (5-j5) + j2 = (5+j7)\text{V}$$





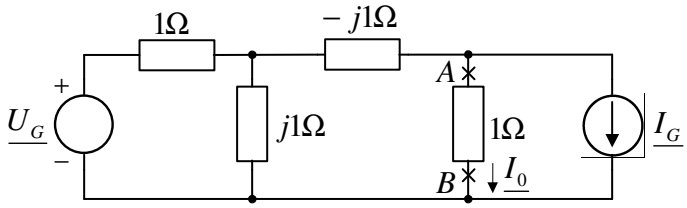
$$\underline{Z}_T = -j + (1 \parallel j) = -j + \frac{j \cdot 1}{j+1} = \frac{1-j}{2}\Omega$$

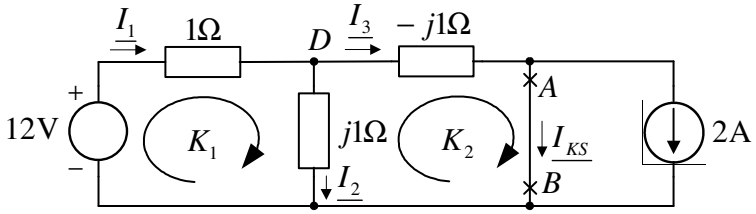


$$\underline{I}_0=\frac{5+j7}{0{,}5-j0{,}5+1}=\frac{5+j7}{1{,}5-j0{,}5}=\left(\frac{8}{5}+j\frac{26}{5}\right)\text{A}$$

$$\underline{I}_0=(1{,}6+j5{,}2)\text{A}=5{,}44\text{A}\angle 72{,}9^\circ \quad\Rightarrow\quad \boxed{i_0(t)=7{,}69\text{A}\cos(10000\pi t+72{,}9^\circ)}$$

e)





Kontura K_1: $\quad 12\text{V}-\underline{I}_1\cdot 1\Omega-\underline{I}_2\cdot j1\Omega=0$

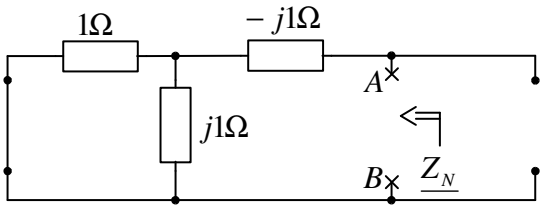
Kontura K_2: $\quad \underline{I}_2\cdot j1\Omega-\underline{I}_3\cdot(-j1)\Omega=0$

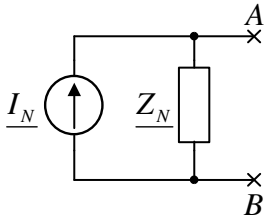
Čvor A: $\quad \underline{I}_3=\underline{I}_{KS}+2\text{A}$

Čvor D: $\quad \underline{I}_1=\underline{I}_2+\underline{I}_3 \quad\Rightarrow\quad \underline{I}_1=\underline{I}_2+\underline{I}_{KS}+2\text{A}$

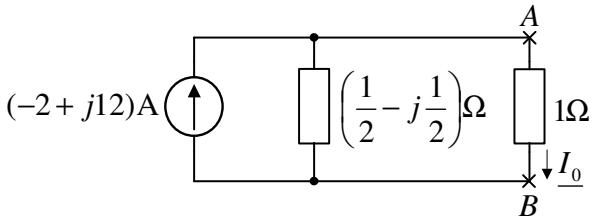
$$\left.\begin{array}{l} 12-\underline{I}_2-\underline{I}_{KS}-2-\underline{I}_2\cdot j=0 \\ \underline{I}_2\cdot j-(\underline{I}_{KS}+2)\cdot(-j)=0 \end{array}\right\} \quad \begin{array}{l} 12+\underline{I}_{KS}+2-\underline{I}_{KS}-2+(\underline{I}_{KS}+2)\cdot j=0 \\ \underline{I}_2\cdot j=(\underline{I}_{KS}+2)\cdot(-j) \quad\Rightarrow\quad \underline{I}_2=-\underline{I}_{KS}-2 \end{array} \Bigg\} \quad \underline{I}_{KS}=-2-\frac{12}{j}$$

$$\underline{I}_N=\underline{I}_{KS}=(-2+j12)\text{A}$$





$$\underline{Z}_N = -j + (1 \parallel j) = -j + \frac{j \cdot 1}{j+1} = \frac{1-j}{2}\,\Omega$$

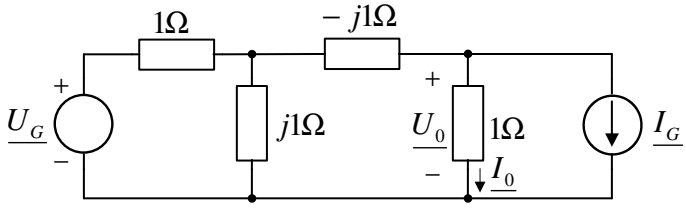


$$\underline{I}_0 = \frac{0{,}5 - j0{,}5}{0{,}5 - j0{,}5 + 1} \cdot (-2 + j12) = \left(\frac{8}{5} + j\frac{26}{5}\right)\text{A}$$

$$\underline{I}_0 = (1{,}6 + j5{,}2)\text{A} = 5{,}44\text{A}\angle 72{,}9° \quad \Rightarrow \quad \boxed{i_0(t) = 7{,}69\text{A}\cos(10000\pi t + 72{,}9°)}$$

48. Za kolo iz prethodnog zadatka odrediti faktor snage, kompleksnu, aktivnu, reaktivnu i prividnu snagu koja se razvija na otporniku kroz koji protiče struja i_0.

Rešenje:



Kompleksna snaga: $\underline{S} = \underline{U}_0 \cdot \underline{I}_0^* = 1\Omega \cdot \underline{I}_0 \cdot \underline{I}_0^* = 1\Omega \cdot \left|\underline{I}_0\right|^2 = 1 \cdot (1{,}6^2 + 5{,}2^2) \quad \Rightarrow \quad \boxed{\underline{S} = 29{,}6}$

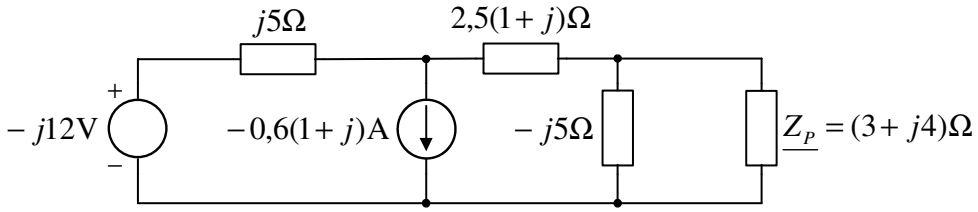
$\underline{S} = P + jQ$

Aktivna snaga: $\boxed{P = 29{,}6\text{W}}$ Reaktivna snaga: $\boxed{Q = 0\text{VAr}}$

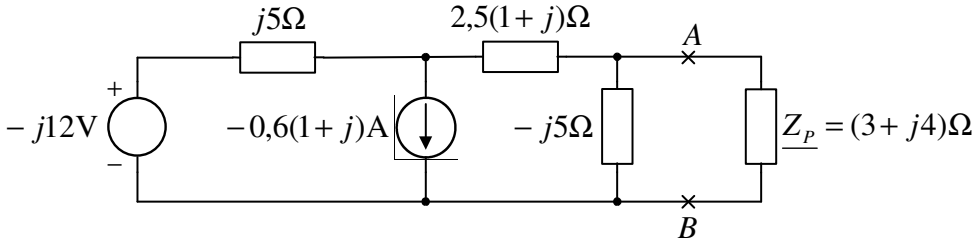
Prividna snaga: $S = \left|\underline{S}\right| = \sqrt{P^2 + Q^2} \Rightarrow \boxed{S = 29{,}6\text{VA}}$

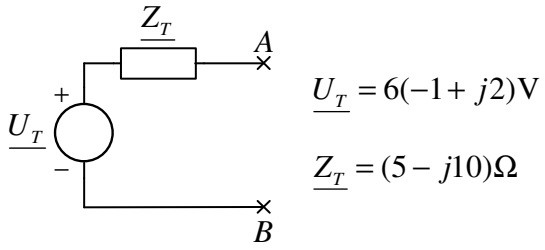
Faktor snage: $\cos\phi = \dfrac{P}{S} \quad \Rightarrow \quad \boxed{\cos\phi = 1}$

49. Za kolo sa slike odrediti faktor snage, kompleksnu, aktivnu, reaktivnu i prividnu snagu koja se razvija na potrošaču $\underline{Z}_P$.



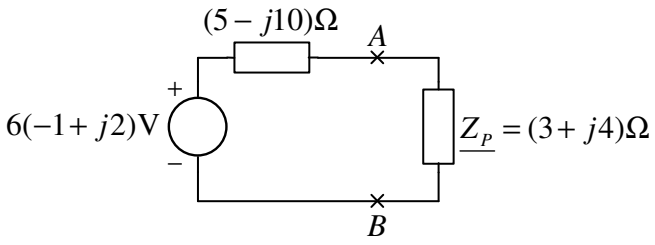
Rešenje:

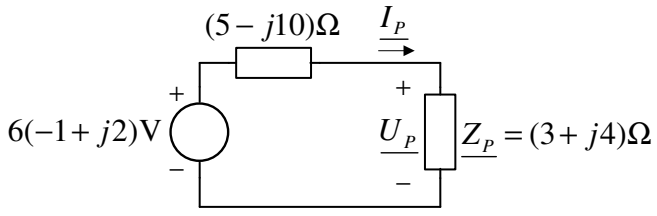




$\underline{U}_T=6(-1+j2)\text{V}$

$\underline{Z}_T=(5-j10)\Omega$





$$\underline{I_P} = \frac{6(-1+j2)\text{V}}{(5-j10)\Omega + (3+j4)\Omega} = \frac{3(-2+j)}{5}\text{A} = (-1{,}2 + j0{,}6)\text{A}$$

Kompleksna snaga:

$$\underline{S} = \underline{U_P}\cdot\underline{I_P^*} = \underline{Z_P}\cdot\underline{I_P}\cdot\underline{I_P^*} = \underline{Z_P}\cdot\left|\underline{I_P}\right|^2 = (3+j4)\cdot((-1{,}2)^2 + 0{,}6^2) \quad \Rightarrow \quad \boxed{\underline{S} = 5{,}4 + j7{,}2}$$

$$\underline{S} = P + jQ$$

Aktivna snaga: $\boxed{P = 5{,}4\text{W}}$

Reaktivna snaga: $\boxed{Q = 7{,}2\text{VAr}}$

Prividna snaga: $S = \left|\underline{S}\right| = \sqrt{P^2 + Q^2} \quad \Rightarrow \quad \boxed{S = 9\text{VA}}$

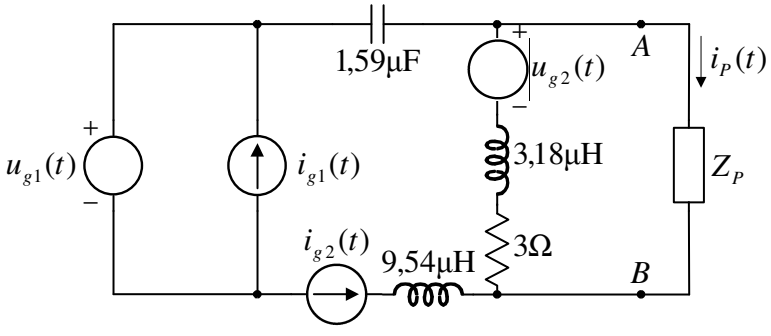
Faktor snage: $\cos\phi = \frac{P}{S} \quad \Rightarrow \quad \boxed{\cos\phi = 0{,}6}$

50. Kolo naizmenične struje sa slike radi u ustaljenom prostoperiodičnom režimu na frekvenciji $f = 50\text{kHz}$. Poznato je da je $u_{g1}(t) = 2\text{V}\cos(2\pi ft + 45°)$, $u_{g2}(t) = -2\sqrt{2}\text{V}\sin(2\pi ft)$, $i_{g1}(t) = 2\text{A}\cos(2\pi ft - 45°)$ i $i_{g2}(t) = -\sqrt{2}\text{A}\sin(2\pi ft)$.

a) Odrediti parametre ekvivalentnog Tevenenovog generatora u kompleksnom domenu za deo kola levo od tačaka A i B.

b) Odrediti elemente (i vrednosti elemenata) koji treba da sačinjavaju potrošač Z_P, tako da se na njemu razvija maksimalna aktivna snaga.

c) Pod uslovom iz tačke b) odrediti struju $i_P(t)$.



Rešenje:

$u_{g1}(t) = 2\text{V}\cos(2\pi ft + 45°) \quad \Rightarrow \quad \underline{U_{g1}} = \sqrt{2} \cdot e^{j45°} = (1 + j)\text{V}$

$u_{g2}(t) = -2\sqrt{2}\text{V}\sin(2\pi ft) \quad \Rightarrow \quad \underline{U_{g2}} = -2 \cdot e^{-j90°} = j2\text{V}$

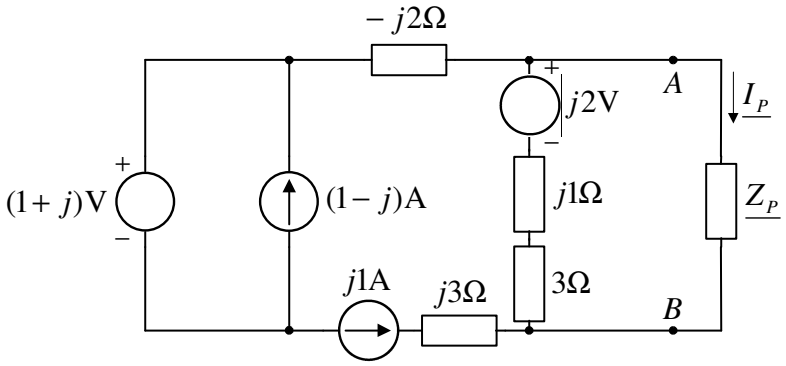
$i_{g1}(t) = 2\text{A}\cos(2\pi ft - 45°) \quad \Rightarrow \quad \underline{I_{g1}} = \sqrt{2} \cdot e^{-j45°} = (1 - j)\text{A}$

$i_{g2}(t) = -\sqrt{2}\text{A}\sin(2\pi ft) \quad \Rightarrow \quad \underline{I_{g2}} = -1 \cdot e^{-j90°} = j1\text{A}$

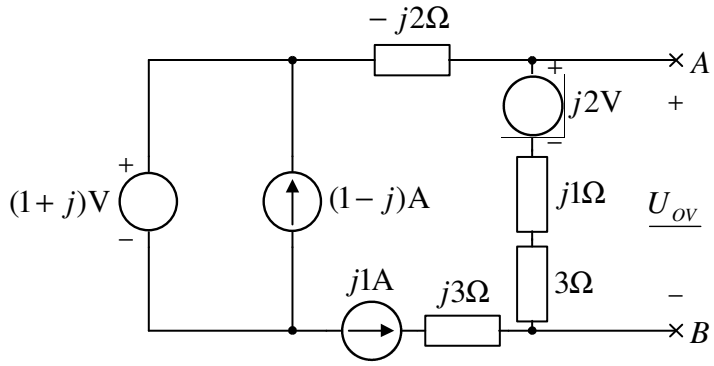
$L_1 = 3{,}18\mu\text{H} \quad \rightarrow \quad \underline{Z_{L1}} = j\omega L_1 = j \cdot 2\pi \cdot 50000\text{Hz} \cdot 3{,}18\mu\text{H} = j1\Omega$

$L_2 = 9{,}54\mu\text{H} \quad \rightarrow \quad \underline{Z_{L2}} = j\omega L_2 = j \cdot 2\pi \cdot 50000\text{Hz} \cdot 9{,}54\mu\text{H} = j3\Omega$

$C = 1{,}59\mu\text{F} \quad \rightarrow \quad \underline{Z_C} = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C} = -\dfrac{j}{2\pi \cdot 50000\text{Hz} \cdot 1{,}59\mu\text{F}} = -j2\Omega$

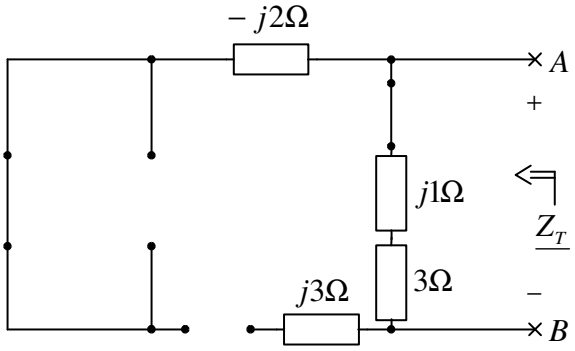


a)

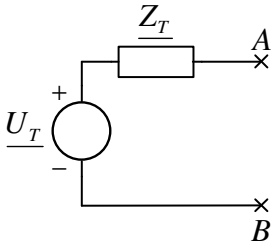


$\underline{U_T} = \underline{U_{OV}} = -j \cdot (3 + j) + j2$

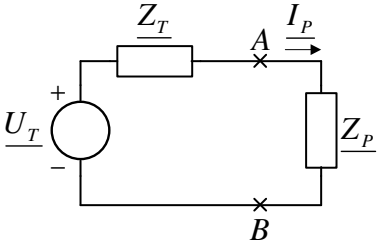
$\boxed{\underline{U_T} = (1 - j)\text{V}}$



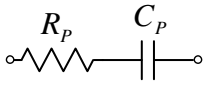
$$\underline{Z_T} = (3 + j)\Omega$$



b)



$$\underline{Z_P} = \underline{Z_T^*} \quad \Rightarrow \quad \boxed{\underline{Z_P} = (3 - j)\Omega}$$



$$\boxed{R_P = 3\Omega} \qquad \boxed{C_P = 3{,}18\mu\text{F}}$$

c) $$\underline{I_P} = \frac{\underline{U_T}}{\underline{Z_T} + \underline{Z_P}} = \frac{1 - j}{6}\text{A} = (\frac{1}{6} - j\frac{1}{6})\text{A} = \frac{\sqrt{2}}{6}\text{A} \cdot e^{-j45°}$$

$$i_P(t) = \frac{1}{3}\text{A}\cos(2\pi ft - 45°) \quad \Rightarrow \quad \boxed{i_P(t) = \frac{1}{3}\text{A}\cos(100000\pi t - 45°)}$$